

“Cosmological” scenario for $A - B$ phase transition in superfluid ^3He .

Yu. M. Bunkov, O. D. Timofeevskaya¹.

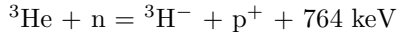
Centre de Recherches sur les Très Basses Températures - CNRS, BP166, 38042, Grenoble, France.
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At a very rapid superfluid transition in ^3He , follows after a reaction with single neutron, the creation of topological defects (vortices) has recently been demonstrated in accordance with the Kibble-Zurek scenario for the cosmological analogue. We discuss here the extension of the Kibble-Zurek scenario to the case when alternative symmetries may be broken and different states nucleated independently. We have calculated the nucleation probability of the various states of superfluid ^3He during a superfluid transition. Our results can explain the transition from supercooled A phase to the B phase, triggered by nuclear reaction. The new scenario is an alternative to the well-known “baked Alaska” scenario.

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Superfluid ^3He has an order parameter which describe the simultaneous spin, orbital and gauge symmetries which are broken at the superfluid transition. This transition can be regarded as the closest condensed matter analogy to the cosmological grand unification transition. This analogy have been utilised in the experimental test of the Kibble cosmological mechanism of cosmic strings creation. According to this mechanism [1], at the transition separate regions of the Universe are independently nucleated with a random orientation of the gauge field in each region. The size of these initial regions (domains) depends strongly on the rapidity with which the transition is traversed. According to Zurek [2] the fundamental distance between the independently-created coherent domains (in the language of [2] the distance between the ensuing vortices Z) is of the order of $Z = \xi_0(\tau_Q/\tau_0)^{1/4}$, where ξ_0 is the zero temperature coherence, $\tau_0 = (\xi_0/v_F)$ is the characteristic time constant of the superfluid, and τ_Q is the characteristic time for cooling through the phase transition. As the domains grow and make contact with their neighbours, the resulting gauge field cannot be uniform. The subsequent order-parameter “glass” forces a distribution of topological defects leading to a tangle of quantized vortex lines. The first quantitative tests of defect creation during a gauge symmetry transformation have been recently performed in superfluid ^3He .

The superfluid ^3He (at very low temperature in the Grenoble experiment [3] and at a relatively high temperature in the Helsinki experiment [4]) was heated locally by neutron irradiation via the nuclear reaction:



The energy released by a neutron reaction heats a small region of the liquid ^3He (about $30\mu\text{m}$) into the normal state. This region recools rapidly through the superfluid transition owing to the rapid outflow of quasiparticles into the surrounding superfluid. For the experimental conditions of both experiments it has been proposed that quasiparticles from the heated region disperse outwards, meaning that the hot bubble is cooled rapidly from its sides and that the cooling rate is so fast that the or-

der parameter of the surrounding superfluid ^3He cannot follow the changing temperature front fast enough (see [5] for theoretical details). Consequently internal regions of the hot volume transit into the superfluid phase independently in accordance with the Zurek cosmological scenario. The experimental results of both experiments justify this assumption. In the Grenoble experiment the excess number of quasiparticles created by the reaction has been counted and it was found that a significant fraction of the energy released by the reaction does not appear in the quasiparticles thermal reservoir. This energy deficit agrees well in magnitude with the energy expected to be trapped as topological defects (in this case vortices) as calculated from Zurek’s scenario for the Kibble mechanism.

Under the relatively high temperature conditions of the Helsinki experiment any vortices created by the neutron reaction would be rapidly destroyed via interaction with the quasiparticle gas. However, in the ROTA project rotating cryostat there is an added bias field, that of rotation. This field can extract a few vortex rings from the bubble which then grow to the dimensions of the cell. After the process the number of vortices can be measured directly by NMR. The number of extracted vortices corresponds well to that calculated from the Zurek scenario.

Our knowledge of superfluid ^3He is much better than our knowledge of the Universe. In the case of superfluid ^3He we not only know the symmetries broken during the superfluid transition but we also know the Ginzburg-Landau potential exactly and we can calculate quantitatively the dynamics of the order parameter during the transition. There are two different stable phases of ^3He , the A and B phases which correspond to different broken symmetries. The energy difference between these two states is relatively small. Let us say that it is negligible on the timescale of the transition! This means that regions which independently enter the superfluid state, should not only have different orientation of the order parameter but may also correspond to states with different symmetries [6]. It is this complication of the Kibble-

Zurek scenario which we considered in the calculations below. Ironically, a very similar situation may be relevant to the Universe, where in addition to the creation of the $SU(3) \times SU(2) \times U(1)$ state, other states may also be created, in particular, $SU(4) \times U(1)$ state [7]. The first state, we believe, corresponds to the energy minimum of our Universe, whereas the second state has much higher creation probability owing to its higher symmetry. This is exactly the situation in superfluid ^3He where the B phase has the lower energy, except in the case of the strong interaction correction for high pressure and temperature.

The rotational and gauge symmetries of ^3He are usually represented by a 3×3 matrix of complex numbers A_{ai} which is known as the order parameter. Above the transition all the elements of the matrix have zero values (representing full symmetry). Below the transition, some of these quantities become non-zero. The symmetry of the order parameter after the transition corresponds to the manifold of symmetries which remain unbroken. In the case of superfluid ^3He there are 13 possibilities (13 states) corresponding to the various symmetries of the order parameter [8]. The free energy of these states can be expressed in the framework of the phenomenological theory of Ginzburg and Landau by:

$$F = -\alpha A_{ai}^* A_{ai} + \beta_1 A_{ai}^* A_{ai}^* A_{bj} A_{bj} + \beta_2 A_{ai}^* A_{ai} A_{bj}^* A_{bj} + \beta_3 A_{ai}^* A_{bi}^* A_{aj} A_{bj} + \beta_4 A_{ai}^* A_{bi}^* A_{bj}^* A_{aj} + \beta_5 A_{ai}^* A_{bi} A_{bj} A_{aj}^*$$

where $\alpha = \alpha_0(1 - T/T_c)$ changes sign at the transition temperature T_c , and the quantities β_i are functions of pressure (and also of temperature through the so-called “strong correction”) and depend on the details of the microscopic interaction.

The different possible symmetries of the order parameter correspond to local minima and saddle points in this 18-dimensional energy surface. In superfluid ^3He we know there are two stable states, the A and B phases. The energy balance between the A and B phases is determined by the relation between the parameters β_i . At zero pressure, the B phase corresponds to the absolute minimum, while at pressures above 20 bar there is a region of temperature where the A phase becomes the preferred state.

These two states have different order parameter symmetries. In the B phase, relative spin-orbit symmetry $SO(3)_{S+L}$ remains unbroken (such that A_{ai} resembles a rotation matrix). In the A phase (the “axial” state) the symmetry of the spin system is reduced to a gauge symmetry, which couples to the orbital motion to yield a combined symmetry of the orbital rotation and gauge fields $U_S \times U_{L+G}$.

According to Zurek scenario, regions on a distance scale of Z undergo the superfluid transition separately. We can consider these regions as independent elementary samples of ^3He . (Later we shall analyse the influence of the gradient energy between the different regions.) We have numerically modelled the creation of the super-

fluid phases in a single region during a rapid superfluid transition. We applied a small random perturbation to the A_{ai} matrix at $T = T_c$. Then we have reduced the temperature with some velocity and have calculated the development of order parameter during this “downhill” process. For this we have applied the time dependent Ginzburg-Landau equation in the form:

$$-\tau \frac{\partial}{\partial t} A_{a,i} + \frac{T_c - T(t)}{T_c} A_{a,i} - (\beta_1 A_{ai}^* A_{bj} A_{bj} + \beta_2 A_{ai} A_{bj}^* A_{bj} + \beta_3 A_{bi}^* A_{aj} A_{bj} + \beta_4 A_{bi} A_{bj}^* A_{aj} + \beta_5 A_{bi} A_{bj} A_{aj}^*) = 0$$

We have monitored both the symmetry of order parameter and the energy during this time-evolution. We have found that both the A and B phases (as well as the axi-planar state at 0 bar, see below) can develop. The final state depends on the starting orientation of the order parameter and the profile of the 18-dimensions potential surface. Other metastable states may develop transiently after the application of an initial perturbation which has the exact symmetry of these states. However the trajectory of A_{ai} in these cases is unstable and any small perturbation away from the final symmetry leads to the more stable A or B states.

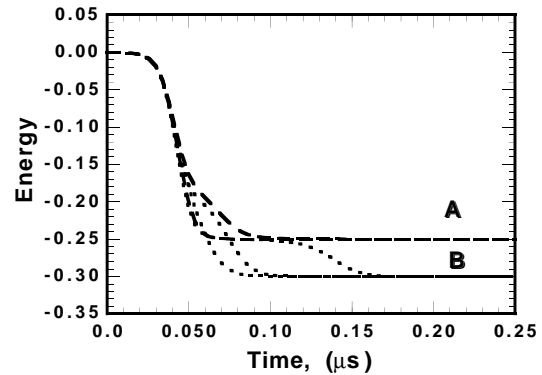


FIG. 1. The time evolution of the free energy density during a superfluid phase transition after a small random perturbations. The temperature was reduced from $T = T_c$ to $T = 0$ in a time of 10^{-8}s . Dashed lines correspond to transitions resulting in A phase, while dotted lines end in B phase.

It is important to note that, although according to Zurek the cooling rate determines the dimensions of the independent regions, the trajectory of the order parameter for a single coherent region is rate independent and is determined only by the profile of the G-L potential. At zero pressure, when we only have the weak interaction where $\beta_i = (-1, 2, 2, 2, -2)$, the B phase corresponds to the absolute energy minimum. In our computer simulation we find that, even under these conditions, nucleation of the A phase has a high probability. In quantitative terms we find the probability of B phase creation to be $54\% \pm 1\%$, while that of the A phase creation is 46% . It is difficult to visualise the trajectory of the order parameter in 18 dimensional space, but we can monitor the G-L energy during the transition. Fig.1 shows typical trajectories of the superfluid ^3He free energy after rapid

cooling. In some cases the trajectory approaches regions of saddle points on the energy surface. The behavior here is clarified by reducing the rate of energy change.

For to study the influence of gradient energy on the development of the order parameter we have considered a one-dimensional spatial sample of Zurek length Z divided into 100 points. We have chosen Z to agree with the Grenoble experiment at zero bar (about $8\xi_0$). Two different perturbations have been applied, one for the first 50 points and the other for the second 50 points. The development of the A_{ai} matrix during the “downhill” process has been calculated at each point, taking into account the gradient energy. The results of these calculations, when a perturbation with A-phase symmetry is applied to one side, and with a B-phase symmetry on the other, are shown in Fig.2. We have found that the boundary between the two different states remains almost stationary during the main part of the “downhill” process. Towards the end of this process the boundary begins to move in the energetically favourable direction. This result looks very natural, since the boundary replacement is determined by the energy difference, and the time dependence of the energy is very similar for the two different symmetries at the beginning of the “downhill” process (see Fig.1).

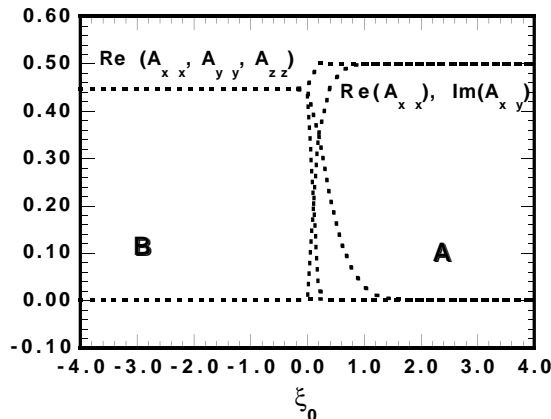


FIG. 2. The nonzero terms of the order-parameter evolution during a superfluid phase transition. At time zero a small perturbation was applied (with B-phase symmetry for the left hand side and A-phase symmetry for the right hand side).

As was discussed by Volovik [9], the A phase at 0 bar has an additional hidden symmetry. It corresponds to the degenerate manifold of states between “axial” state and “planar” state. The planar state corresponds to saddle point, it does not separated by potential barrier from the B state. Nevertheless the domain boundary forms even between “planar” and B states, but then moved relatively fast. With pressure the degeneracy removed in favour of “axial” state.

The β_i parameters depend on the pressure and temperature. There are a number of theories which suggest

somewhat different dependencies of these parameters on pressure at T_c . We have used the parameters, calculated by Sauls and Serene [10]. In Fig.3 we show the probability of A phase nucleation as a function of pressure along with the energy balance between the A and B phases. It is important to notice that the probability of A state nucleation may become greater than 50% even in the region where the B phase is stable.

The temperature dependence of β_i has not been much investigated theoretically. Qualitatively, we expect that they should change in the same way as the A-B equilibrium line changes on the ^3He phase diagram. This would imply that with cooling the strong interaction correction should decrease very rapidly. The temperature dependence of β_i parameters is in fact very important for our scenario, because under the non-equilibrium conditions of the fast phase transition the temperature changes faster than the order parameter.

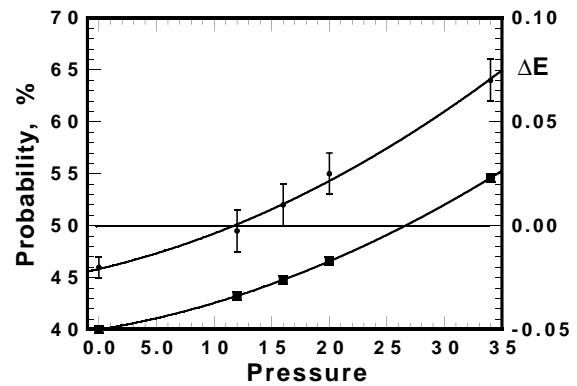


FIG. 3. The probability of A state nucleation as function of pressure for temperature near T_c , and the difference of energy between A and B states.

All experimentalists who work with superfluid ^3He have noticed the crucial asymmetry of the A – B transition. If one is cooling ^3He at a pressure above 20 bar, the A phase may survive as a supercooled metastable state far below the equilibrium A – B transition line. On the other hand, on warming it is difficult to get superheated B phase. In [11] it was shown that a transition from A to B phase will always occur at some critical temperature. The pressure dependence of this threshold temperature is parallel to the equilibrium A-B transition line. It crosses the T_c temperature line at about 15 bar. This corresponds well to the situation in our calculations where the probability of B nucleation exceeds that of A phase nucleation.

This observation may supply the critical jigsaw piece of information for the long-running puzzle of the A – B transition in superfluid ^3He . As proposed by Leggett and demonstrated in the Stanford experiments (see review [12]) cosmic rays can trigger the transition from supercooled A phase to B phase. In the well-known “baked

Alaska” scenario, proposed by Leggett [12], it is assumed that a shell of normal liquid expands from the reaction site. After the shell has passed the temperature inside falls below T_c and a new state nucleates. The presence of the expanding normal shell is needed to isolate the nucleation of a new phase from any influence of the surrounding A phase. From our point of view, this is a rather artificial suggestion. It is likely that the cosmic event creates very energetic quasiparticles. These energetic quasiparticles travel out from the site of the event and create many new low energy quasiparticles on thermalization. It is important to point out that the low energy quasiparticles do not maintain the direction of the primary energetic ones. That is why it is likely that the quasiparticles remain inside the hot bubble and expand by the usual diffusion process.

However, in framework of the cosmological Kibble-Zurek approach we do not need such a normal shell to protect the interior of the hot bubble from the influence of the outside state. The diffusion cooling runs so fast that many seeds of A and B phase are nucleated independently. The “backed Alaska” process, if it occurs, would lead to an even larger number of such domains. The subsequent development of the structure depends first on the relative densities of the two phases and secondly on the energy balance between them and on the domain boundary surface energy. If one state has a significantly higher probability of nucleation than the other, then percolation occurs and the more probable phase grows at the expense of the less probable to reduce the surface energy. That is the reason for the asymmetry in the $A - B$ transition. On cooling, the A phase can be supercooled because after a cosmic rays events the seeds of B state do not survive in conditions where the A phase has a higher nucleation probability. For to pass throw transition, the seeds of B states should form a cluster of critical dimensions, which is possible only when B state nucleation probability is near to 50%.

In the case where there is a possibility of nucleating two distinct phases, then owing to the eventual suppression of one phase, the distance between the subsequent vortices which remain from the order-parameter “glass” will be larger than that implied by the straightforward Zurek scenario. A simple argument suggests that the separation increases by of order $Q^{-0.5}$, where Q is the probability of nucleation of the surviving state. This correction makes the calculated distance between vortices closer to that observed in the Grenoble [3] experiment. Furthermore, the influence of the proximity of the A phase on the den-

sity of vortices created has recently been demonstrated [13]

Having considered superfluid ^3He we should look more carefully at similar possibilities for the early Universe. In other words, the vacuum of the Universe after a grand unification transition may also have had metastable states with different symmetries. For example vacua with symmetries $(\text{SU}(4) \times \text{U}(1))$ and $(\text{SU}(3) \times \text{SU}(2) \times \text{U}(1))$ might have been able to coexist in the early Universe in separate domains. The spatial scale of these domains should be of the order of the parameter Z in Zurek’s scenario. The transition of the metastable phase to the stable might have given rise to temperature and density inhomogeneities which may have influenced the Universe inhomogeneity observed at present.

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- ¹ Permanent address: Moscow State University, Bogoliubov Institute of Theoretical Microphysics, Russia.
- [1] T. W. B. Kibble, *J. Phys.* **A9**, 1387 (1976).
 - [2] W. H. Zurek, *Nature* **317**, 505 (1985).
 - [3] C. Bauerle, Yu. M. Bunkov, S. N. Fisher, H. Godfrin, G. R. Pickett, *Nature* **382**, 332 (1996).
 - [4] V. M. H. Ruutu, V. B. Eltsov, A. J. Gill, T. W. B. Kibble, M. Krusius, Yu. G. Makhlin, B. Plaçais, G. E. Volovik, Wen Xu, *Nature* **382**, 334 (1996).
 - [5] T. W. B. Kibble, G. E. Volovik, *JETP letters* **65**, 96 (1997).
 - [6] G. E. Volovik have been first, who point out this circumstances, in unpublished version of [4].
 - [7] A. Linde, *Particle physics and inflationary cosmology* (Harwood acad. pbl., Switzerland, 1990).
 - [8] D. Vollhardt, P. Wolfle, *The Superfluid Phases of ^3He* (Taylor and Francis, London, 1990).
 - [9] G. E. Volovik, *Exotic Properties of Superfluid ^3He* (World Scientific, Singapore, 1992).
 - [10] J. A. Sauls, J. W. Serene, *Phys. Rev.* **B24**, 183 (1981).
 - [11] P. J. Hakonen, M. Krusius, M. M. Salomaa, J. T. Simola, *Phys. Rev. Lett.* **54**, 245 (1985).
 - [12] P. Schiffer, D. D. Osheroff, A. J. Leggett, in *Progress in Low Temperature Physics* v. 14, ed. by W. P. Halperin, (Elsevier, Amsterdam, 1995).
 - [13] V. M. Ruutu, V. B. Eltsov, M. Krusius, Yu. G. Makhlin, B. Plaçais, G. E. Volovik, *Phys. Rev.Lett.*, submitted.